# Quasi-Periodic Route to Chaos in a Microwave Doubler

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***Abstract*—Quasi-periodic route to chaos was observed as the bias level was increased in a 5–10 GHz frequency doubler. The appearance of spurious oscillations is a consequence of the dynamical negative resistance manifested by the pn junction diode in the doubler. This phenomenon is caused by the long minority carrier recombination time of the junction diode compared to the period of the input signal.**

## I. Introduction

VARACTOR frequency doublers based on the pn junction diode nonlinearity are well known for their notorious instabilities. In this work, we characterize these instabilities for the first time in the light of chaos theory and explain the physical basis for such phenomena. This characterization is significant since it paves way for the prediction of these instabilities using a structured mathematical approach such as bifurcation theory.

Chaos has been observed in various low frequency circuits and mechanical systems [1] and lately in microwave circuits such as Gunn osciallators [2] and limiters [3]. The path of chaos has been observed to be period doubling, period adding, intermittent, or quasi-periodic routes depending on the system [4]. In the doubler case, only the quasi-periodic route is observed.

The pn junction diode used in the paper is an Alpha DVA6735-06 silicon varactor diode. The doping profile is typical of an abrupt junction diode (doping density/cm$^3$ of $10^{21}$ for $p^+$ diffused, $10^{16}$ for n epi-layer and $10^{19}$ for $n^+$ substrate). It is important to note that the diode was not operated in a Gunn-or Impatt-type modes and the oscillation observed here is largely due to charge storage at the junction, rather than transferred electron, avalanche, or parametric-type effects which may be theoretically possible. The performance of the doubler can be enhanced by using faster varactor diodes (this has an inherent physical limitation), or majority carrier devices such as Schottky barrier or heterostructure diodes.

## II. Chaotic Behavior

The doubler circuit depicted in Fig. 1 is composed of two filters and a pn junction diode. Although it is not shown in the diagram, the bias line has a narrowband RF rejection at the input frequency; hence, it is open to the perturbation effects at other frequencies. The input signal to the doubler was held fixed at 16 dBm and the bias was varied from −3.0 to 0.0 V. At the low end of the bias, the doubler is in normal mode of operation, and no bifurcation occurs; only two spectral lines are visible, that of the fundamental and the second harmonic. As the bias level is increased to −2.59 V, the doubler exhibits a Hopf bifurcation and the oscillation mixes with the input frequency to result in a quasi-periodic regime as depicted in Fig. 2(a). Increasing the bias level further to −1.33 V, the regime bifurcates again to three-frequency quasi-periodic state as shown in Fig. 2(b). This level persists, although vulnerable to external perturbations, until at 0.33 V when the three-frequency regime breaks down to chaos exhibited in Fig. 2(c). The chaotic state is manifested by a broadband noise spectrum around the mixing product spectral lines. The broadband noise spectrum is considered a standard proof of a chaotic state [4]. The chaotic window spans upto a bias level of −0.19 V when the two-frequency quasi-periodic regime reappears [Fig. 2(d)]. These unstable realms are not immune to perturbations. Indeed, chaotic states can be perturbed to a quasi-periodic state and vice versa. Physically, the circuit can be perturbed by merely touching the bias lines, which are designed to be narrowband, thus momentarily providing RF feedback for a broad band of frequencies except at the fundamental of the input signal.

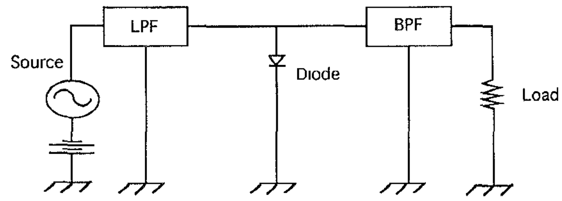


Fig. 1. Microwave frequency doubler circuit composed of lowpass (LPF) and bandpass (BPF) filters and a pn junction diode.

## III. Explanation of Chaos

Hunt's [5] explanation of the occurrence of chaos in a simple RLC driven circuit containing a pn junction diode as its nonlinear element has been confirmed through numerical computations of state equations [6]. The chaos in his circuit was preempted by the period doubling route and follows a classic bifurcation tree pattern, which can be generated through simple logistic equations. A similar explanation holds for the chaos we have observed in the microwave doubler, although here due to a different linear circuitry the chaotic route happens to be quasi-periodic unlike the period doubling observed in [5].

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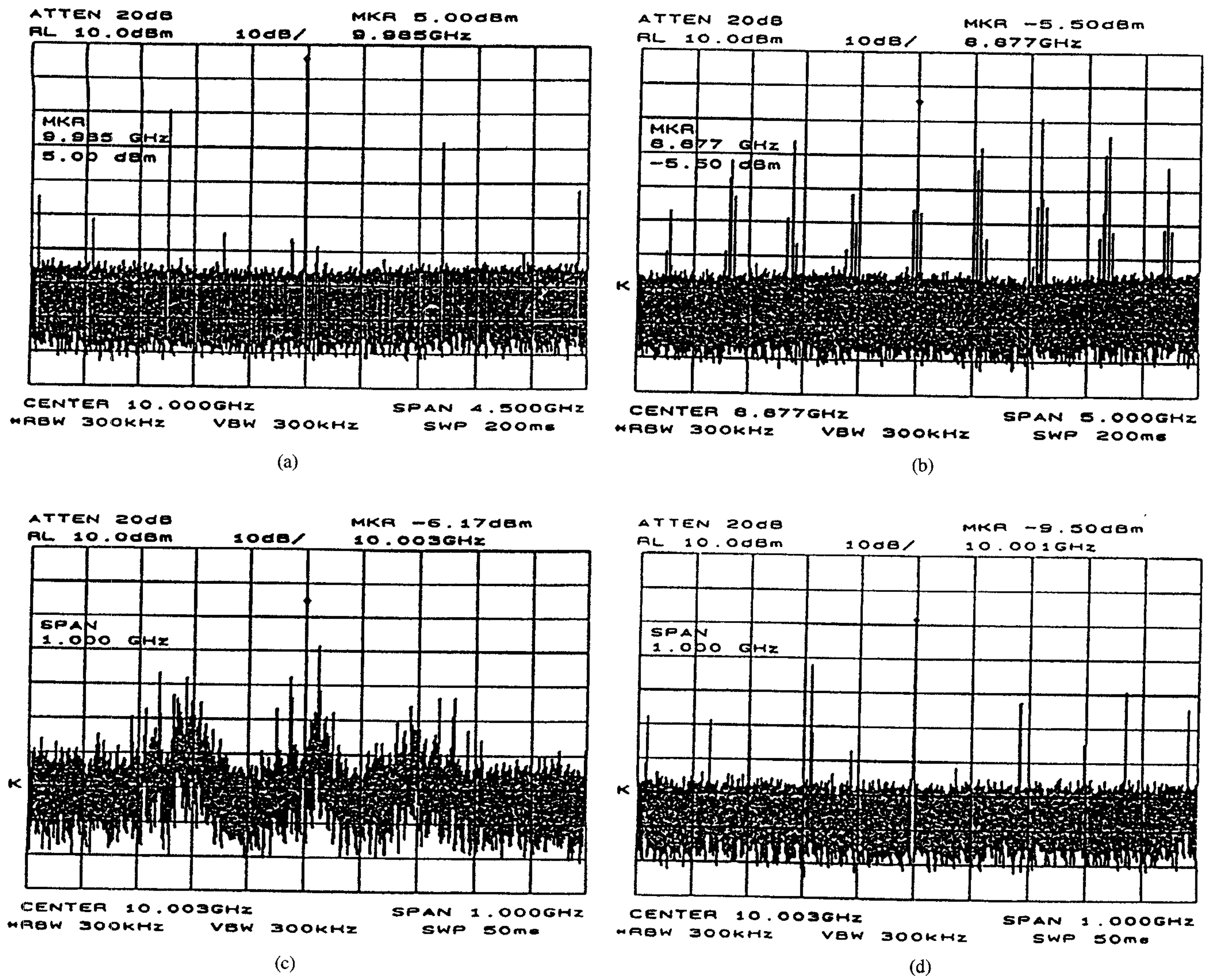


Fig. 2. Spectral portrait of frequency doubler in different states at different bias levels: (a) two-frequency quasi-periodic at $V_b = -2.59$ V, (b) three-frequency quasi-periodic at $V_b = -1.33$ V, (c) chaos at $V_b = -0.33$ V, and (d) two-frequency quasi-periodic at $V_b = -0.19$ V.

The pn junction diode has a finite recombination lifetime. When the diode is switched from the forward biased to the reverse biased mode, it still conducts in the same direction due to the presence of minority carriers which do not have enough time to recombine since it is longer than the period of the input signal. This delay gives rise to a dynamical negative resistance, which grows with increased bias level. The negative resistance sets up a regime where oscillation occurs at more than one frequency. The recombination time must be longer than the input signal period to initiate Hopf bifurcation. This fact has been experimentally tested in [7] using fast signal diodes, where no bifurcation was observed. In addition, it is well known that doublers based on majority carrier nonlinear devices such as Schottky barrier diodes do not exhibit bifurcations. The input signal and the oscillation frequency sets up a two-frequency quasi-periodic regime. With increased bias level another Hopf bifurcation follows resulting in a three-frequency quasi-periodic regime. At this stage, chaos becomes likely to occur from external perturbations. This fact has been mathematically proven for generic systems and experimentally observed by Ruelle and Takens [8]. In phase space, a two-frequency system is described by a torus, where the inner and outer radii denote the two frequencies. According to the Ruelle and Takens scenario, the torus may breakdown to chaos or may metamorphize to a three-frequency quasi-periodic regime before chaos depending on the system. In our case, the latter phenomenon has been observed.

Subsequent experimental and numerical work has been done on the doubler to avoid these instabilities. One method is to have a back-to-back series diode structure, which is always reverse-biased and prevents charge storage at the junction. Another method is to operate the diode at a bias level low enough to prevent the total combined voltage (bias level and the input RF level) across the diode from exceeding zero volt.

## IV. Conclusion

We have characterized the spurious oscillations in the microwave doubler circuit to be a quasi-periodic route to chaos as the system parameter (bias level) is varied. The onset of oscillation is caused by the dynamical negative resistance, which is a result of the relatively slower recombination rate of the minority carriers compared to the input RF signal. Since the instabilities are undesirable for the microwave engineer, we are currently pursuing possible numerical methods to predict such occurrences based on bifurcation theory.

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